Applied General Topology
© Universidad Politécnica de Valencia
Volume 7, No. 1, 2006
pp. 109-124

Making group topologies with, and without, convergent sequences*

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Abstract.
(1) Every infinite, Abelian compact (Hausdorff) group K admits $2^{|K|}$-many dense, non-Haar-measurable subgroups of cardinality $|K|$. When K is nonmetrizable, these may be chosen to be pseudocompact.
(2) Every infinite Abelian group G admits a family $\mathcal{A}$ of $2^{2^{|G|}}$-many pairwise nonhomeomorphic totally bounded group topologies such that no nontrivial sequence in G converges in any of the topologies $\mathcal{T} \in \mathcal{A}$. (For some G one may arrange $w(G, \mathcal{T}) < 2^{|G|}$ for some $\mathcal{T} \in \mathcal{A}$.)
(3) Every infinite Abelian group G admits a family $\mathcal{B}$ of $2^{2^{|G|}}$-many pairwise nonhomeomorphic totally bounded group topologies, with $w(G, \mathcal{T}) = 2^{|G|}$ for all $\mathcal{T} \in \mathcal{B}$, such that some fixed faithfully indexed sequence in G converges to 0_G in each $\mathcal{T} \in \mathcal{B}$.

2000 AMS Classification: Primary: 22A10, 22B99, 22C05, 43A40, 54H11. Secondary: 03E35, 03E50, 54D30, 54E35.

Keywords: Haar measure, dual group, character, pseudocompact group, totally bounded group, maximal topology, convergent sequence, torsion-free group, torsion group, torsion-free rank, p-rank, p-adic integers.

1. Introduction

1.1. **Historical background.** Not long after E. Čech and M. H. Stone associated with each Tychonoff space X its maximal compactification $\beta(X)$ (the so-called Stone-Čech compactification), it was noted, denoting by ω the countably infinite discrete space, that $\beta(\omega)$ contains no nontrivial convergent sequence. This observation stimulated Efimov [10] to pose in 1969 a question which in its full generality remains unsolved today: Does every compact Hausdorff space

* Portions of this paper were presented by the first-listed author at the 2004 Annual Meeting of the American Mathematical Society (Phoenix, January, 2004).